

BIFURCATION THEORY AND APPLICATIONS

1. Dynamics and bifurcations

- Topological equivalence of dynamical systems
- Bifurcations and bifurcation diagrams. Classifications. The codimension of a bifurcation. Structural stability
- Topological normal forms for bifurcations
- Versal deformations of matrices
- Center manifolds
- Some models from biology and economy

2. Bifurcations of fold singularities

- Normal form of the fold bifurcation. Fold bifurcation theorem
- Computation of the center manifold
- Pitchfork bifurcation
- Cusp bifurcation

3. Hopf bifurcations

- Normal form of the Hopf bifurcation. Hopf bifurcation theorem
- Computation of the first Liapunov coefficient
- Computation of the center manifold
- Bautin (generalized Hopf) bifurcation. Normal form. An approximation of the parameters curve corresponding to fold bifurcation of cycles.

4. Homoclinic bifurcations

- Homoclinic bifurcations in planar systems. Andronov-Leontovich theorem. Saddle-node homoclinic bifurcation. Double homoclinic bifurcation
- Homoclinic bifurcations in n -dimensional dynamical systems, $n > 2$. Shilnikov theorems.

5. Bogdanov-Takens bifurcation

- Bogdanov normal form
- Takens normal form. Topological equivalence between Bogdanov normal form and Takens normal form
- An approximation of the parameters curve corresponding to homoclinic bifurcation

6. Heteroclinic bifurcations

7. Global bifurcation diagram

8. Numerical analysis of bifurcations

- The software WINPP (XPPAUT)
- Phase dynamics using WINPP
- Parametric portrait using the package LOCBIF from WINPP
- Applications

References

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