UNIVERSITY OF CRAIOVA
Faculty of Mathematics and Computer Science
Fundamental domain : Exact sciences
Domain: Mathematics
Speciality: mathematics
Duration of studies: 2 years
Master: Applied mathematics
Approved with academic year 2009-2010

Spectral Theory of Differential Operators
Syllabus

Course Coordinator: Asist. dr. Mihai Mihailescu
Cod : MA 211
Cycle II : Master
Second Year , Semester I, Cours 28 hours, Seminar 28 hours
No. of credits : 6
Domain: mathematics
Type : mandatory
Category: fundamental

Objectives: The presentation of fundamental results concerning the spectrum of differential operators: the spectrum of the Laplace type operators (the linear case), the spectrum of p-Laplace type operators (the nonlinear case) and the spectrum of p(x)-Laplace type operators (the nonlinear and nonhomogeneous case).

Prerequisites: Functional Analysis, Nonlinear Analysis, Partial Differential Equations.

Evaluation form : Exam(E) : Written exam (and facultatively a viva voce exam) consisting in a test of 1-2 hours with theoretical and practical questions totalizing 9 points (plus one more point given to all the students at the start of the exam).

Contents:

The spectrum of the Laplace operator: Spectrum description. The simplicity of the first eigenvalue. The variational characterization of the first eigenvalue.
The spectrum of the p-Laplace operator: Spectrum description. The simplicity of the first eigenvalue. The variational characterization of the first eigenvalue.
The spectrum of the Laplace and p-Laplace type operators with weight.
Spectral problems concerning p(x)-Laplace type operators: The definition of Lebesgue and Sobolev variable exponent spaces and basic properties. The study of spectrum for different problems concerning p(x)-Laplace type operators. The comparation with the results obtained in the case when p(x)=constant.
Spectral problems concerning nonhomogeneous operators on Orlicz-Sobolev spaces: The definition of Orlicz spaces. The main properties of Orlicz spaces. The study of the spectrum for problems involving nonhomogenous differential operators.

Bibliography:

1. W. Allegretto, *Principal eigenvalues for indefinite-weight elliptic problems in* $\mathbf{R}^n$, Proceedings of the American Mathematical Society **116**, Vol. 3, 1992, 701-706.
2. A. Anane, *Simplicité et isolation de la première valeur propre du p-laplacien avec poids*, C. R. Acad. Sci. Paris, **305**, Série I, 1987, 725-728.
3. H. Brezis, *Analyse fonctionelle. Théorie et applications*, Masson, Paris, 1992.
4. E. B. Davies, *Spectral Theory and Differential Operators*, Cambriedge University Press, 1995.
5. Y. Egorov and V. Kondratiev, *On Spectral Theory of Elliptic operators*, Birhauser, Berlin, 1996.
6. A. Henrot, *Extremum Problems for Eigenvalues of Elliptic Operators*, Birhauser, Basel, 2006.

7. M. Mihailescu and V. Radulescu, *On a nonhomogeneous quasilinear eigenvalue problem in Sobolev spaces with variable exponent*, Proceedings Amer. Math. Soc. **135** (2007), 2929-2937.
8. M. Mihailescu and V. Radulescu, *Spectrum in an unbounded interval for a class of nonhomogeneous differential operators*, Bulletin of the London Mathematical Society **40** (6) (2008), 972-984.
9. A. Szulkin and M. Willem, *Eigenvalue problems with indefinite weight*, Studia Mathematica **135** (2) (1999), 191-201.
10. M. Willem, *Analyse harmonique réelle*, Birhauser, Basel, 1996.