UNIVERSITY OF CRAIOVA
Faculty of Mathematics and Computer Science
Department of Mathematics
Fundamental domain : Exact sciences
Domain: Mathematics
Master: Applied mathematics
Form: Day classes
Duration of studies: 2 years
Academic year 2008-2009

Special Topics in Functional Analysis
Syllabus

Course coordinator: Prof. dr. Constantin P. Niculescu
Code: MA112
Second Cycle: MASTER
First Year, Semester 1, Course 28 hours, Seminar 28 hours
No. of credits: 6
Domain: Mathematics
Type: compulsory
Category: fundamental

Objectives: The presentation of some of the most basic results of functional analysis together with their applications to convex analysis, partial differential equations and nonlinear analysis.
Necessary background: Linear algebra, real and complex analysis, elements of functional analysis.
Evaluation: Exam (E).

Contents:

1. **Banach spaces and linear operators**. Basic results. The principle of uniform boundedness. The inversion of bounded linear operators. Compact operators. Integral operators.
2. **Hilbert spaces and orthogonal expansions**. Bases in Hilbert spaces. Orthogonal projections. The adjoint of an operator. Diagonalization of compact self-adjoint operators. Applications to the Sturm-Liouville problems.
3. **Hahn-Banach Theorem. Consequences**: Convex sets. The extension of continuous linear functionals. Locally convex spaces. Weak topologies. Alaoglu-Bourbaki theorem. Separation of convex sets. Reflexive spaces. The duality of the spaces L^p with $1<p<\infty$.
4. **Nonlinear functional analysis**. Differential calculus in Banach spaces. Elements of convex analysis in Banach spaces. The Brouwer and Schauder fixed point theorems.
5. **Applications to partial differential equations**. Fourier transform. Sobolev spaces. Applications to Dirichlet and Neumann problems.

Bibliography:
1. H. W. Alt, *Lineare Funktionalanalysis*, Springer-Lehrbuch, Berlin, 1992.
2. C. P. Niculescu, *Probleme speciale de analiză funcțională*, Ed. Universitaria, Craiova, 2005.
3. W. Rudin, *Analyse fonctionelle*, Ed. Ediscience International, 1995.
4. K. Yosida, *Functional Analysis*, 5th ed., Springer-Verlag, Berlin, 1995.
5. M. Willem, *Analyse fonctionnelle élémentaire*, Ed. Cassini, Paris, 2003.